

Coupling of Crop Assignment and Vehicle Routing for Harvest Planning in Agriculture

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Abstract—A method for harvest planning based on the coupling of crop assignment with vehicle routing is presented. Given multiple fields (up to hundreds), a path network connecting fields, multiple depots at which a number of harvesters are initially located, the main question addressed is: which crop out of a set of different crops to assign to each field. It must be answered by every farm manager at the beginning of every yearly work-cycle starting with plant seeding and ending with harvesting. Rather than solving a pure assignment problem, we also account for connectivity between fields. In practice, fields are often located distant apart. Traveling costs of machinery and limited harvesting windows demand optimized operation and route planning. The proposed method outputs crop assignment to fields and simultaneously determines an optimized sequence in which to service fields of the same crop during harvest. The described scenario is of particular relevance for larger farms and groups of farms that collaborate and share machinery. We derive integer programming (IP) based exact algorithms. For large numbers of fields, where exact algorithms are not tractable anymore, elements of clustering and the solution of local Traveling Salesman Problems (TSP) are added, thereby rendering the method heuristic, but also large-scale applicable.

Index Terms—Logistics, Assignment Problem, Vehicle Routing, Integer Programming, Agriculture.

I. INTRODUCTION

Agriculture is a diverse field ranging from biotech to autonomous robots and finance. At its core, it is related to logistics and intelligent transportation systems. According to [1], there are four main functional areas for the agri-food supply chain: production, harvesting, storage and distribution. This paper focuses on model-based production planning. In fact, in view of recent plunges of agricultural commodity prices [2], that threaten the sustainability of not few farmers, efficiency improvements in production are more important than ever to minimize unnecessary costs. The decision on the assignment of crops to fields is crucial in that it determines the complete yearly work-cycle. In common practice today, crops are manually clustered according to geographical location and often selected accounting for crop rotation [3] (for reducing soil erosion and increasing soil fertility). The spatial clustering is done for faster harvesting. A trend among farmers in Europe is to collaborate in form of limited companies for sharing of machinery. Not seldomly conflicts arise about the sequence in which to harvest multiple fields of identical crops but various owners. This paper is motivated by providing remedy to both the currently as wide-spread and approximate as crucially important practice of crop assignment and the aforementioned conflicts between collaborating farmers by providing a structured methodology.

The basic multiple Traveling Salesman Problem (mTSP) describes the objective of finding total tour cost-minimizing routes for m salesmen that all start and end at a single depot, and all vertices are visited once by exactly one salesman, see [4]. Nonnegative edge cost can refer to, e.g., monetary, space or time units. When accounting for various demands at each vertex and limiting the capacity of vehicles (salesmen), the problem is referred to as the capacitated Vehicle Routing Problem (VRP). Variations include the VRP with time windows, with backhauls and with pickup and delivery, see [5]. The applications are manifold. For vehicle routing with real-time informations, see for example [6] and the references therein. Recently, there has been increased interest in applying logistical optimization in agriculture for scheduling, routing and fleet management [7], [8], [9], [10], [11]. Special focus was on the *coordination* of machinery teams distinguishing between primary (harvester) and service (transport) units referred to as PUs and SUs, see [12], [13], [14], [15]. All of these references assume that fields with assigned crops are given. To the best of the author's knowledge, the optimized assignment of crops to fields and *simultaneously* accounting for vehicle routing and other constraints for optimized harvest planning has not been discussed in the literature. We propose such strategic assignment to be conducted once per year and at the *beginning* of the yearly work-cycle, thereby decisively affecting the complete yearly agricultural production-cycle, as the first step within a two-layered framework. The second layer involves coordinations of PUs and SUs exploiting all of the aforementioned references, and is to be conducted at the *end* of the yearly work-cycle.

The contribution of this paper is a novel method that can assist farm managers in strategic planning of crop assignment to available fields and simultaneously outputting sequences for harvesting corresponding fields. Special emphasis is on generality admitting to formulate a variety of tailored integer linear programs. The approach can be used in related problems (not necessarily agricultural) coupling assignment and routing.

This paper is organized as follows. The problem and notation is formulated in Section II. Integer linear programs are discussed in Section III. A numerical simulation example is given in Section VI, before concluding with Section VII.

II. PROBLEM FORMULATION AND NOTATION

A. Problem Formulation

For optimized harvest planning in agriculture, we consider four key infrastructural components illustrated in Figure 1. See also Figure 2 for problem visualization. We pose four interrelated questions.

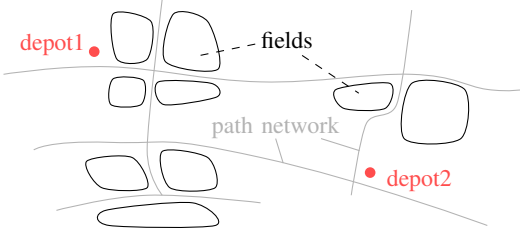


Fig. 1. Visualization of three key components for planning: multiple fields, a path network connecting the fields, and multiple depots. At each depot, multiple harvesters (the fourth key component) may initially be based.

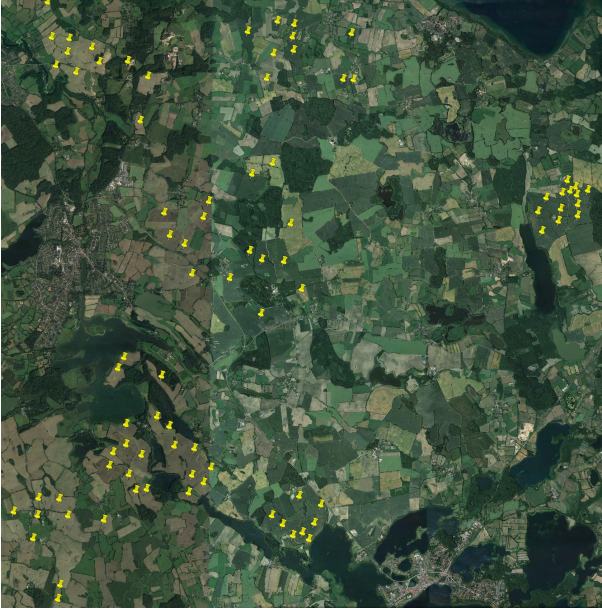


Fig. 2. Problem visualization. Yellow markers indicate fields to be served by collaborating farms. Overall, there are 85 fields. The satellite image shows an area of 15.9×16.3 km. The path network connecting the fields is curvy and often along rural gravel roads only permitting slow traveling speeds. The overall field coverage area is more than 1700ha. Traveling distances between pairs of fields is between meters up to dozens of km.

Problem 1. Every year, at the beginning of the yearly work-cycle, a crop has to be assigned to all available fields.

- 1) Which crop to optimally assign to each field?
- 2) In what sequence to optimally service all fields during harvest?
- 3) How to optimally dispatch multiple harvesters that initially may be located at multiple depots to the multiple fields?
- 4) Which fields should be serviced, and which leased instead, and at what prices?

The first question decides the complete yearly work-cycle of the farm. For optimized harvest planning, its answer must simultaneously account for questions 2) to 4).

We stress that at the beginning of every yearly work-cycle of a farm, planning decisions according to Problem 1 have to be taken. In this paper, we therefore derive optimization problems that permit to input parameters such as, e.g., yields per field and crop. The selection of these parameters largely determines the output of the optimization, and should be based on a mod-

eling step involving historical data (experience). Then, at the end of every yearly work-cycle, i.e., at harvest, deviations from initial modeling have occurred. For example, the actual amount of crop harvested per field is different from predicted, and weather is influencing potential harvesting-windows. Thus, at the end of the yearly work-cycle, the aforementioned second framework-layer becomes relevant (involving the coordination of PUs and SUs). In this paper, we exclusively focus on the first framework-layer.

B. Notation

Let us introduce notation mainly adopting [5]. We denote a complete graph $G = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V} = \{0, \dots, D-1, D, \dots, D+L-1\}$ and \mathcal{A} are vertex and arc set, respectively. The cardinality of a set of vertices is denoted by $|\cdot|$. Vertices $i \in \mathcal{D} = \{0, \dots, D-1\}$ and $i \in \mathcal{L} = \{D, \dots, D+L-1\}$ correspond to D depots and L fields. The corresponding geographical coordinates are denoted by $P_l = (X_l, Y_l) \in \mathbb{R}^2$, $\forall l \in \mathcal{L}$, and similarly P_d , $\forall d \in \mathcal{D}$. The K difference crops are indexed by $\mathcal{K} = \{0, \dots, K-1\}$. Let the number of harvesters located at a depot and suitable for a crop be denoted by $N_d^{\text{harv},k}$, $\forall d \in \mathcal{D}$, $\forall k \in \mathcal{K}$. Let the normalized traveling cost per harvester and crop k between a depot d and a field j or between two fields i and j be denoted by \tilde{c}_{dj}^k and \tilde{c}_{ij}^k , respectively. Abbreviating $N^{\text{harv},k} = \sum_{d \in \mathcal{D}} N_d^{\text{harv},k}$, we define traveling costs as follows:

$$c_{ij}^k = N^{\text{harv},k} \tilde{c}_{ij}^k, \quad \forall i, j \in \mathcal{L}, \quad \forall k \in \mathcal{K}, \quad (1)$$

$$c_{dj}^k = N^{\text{harv},k} \tilde{c}_{dj}^k, \quad \forall d \in \mathcal{D}, \quad \forall j \in \mathcal{L}, \quad \forall k \in \mathcal{K}, \quad (2)$$

$$c_{dj}^{k, \min} = \sum_{\tilde{d} \in \mathcal{D}} N_{\tilde{d}}^{\text{harv},k} \tilde{c}_{\tilde{d}j}^k, \quad \forall j \in \mathcal{L}, \quad \forall k \in \mathcal{K}. \quad (3)$$

Furthermore, we define c_{jd}^k and $c_{jd}^{k, \max}$ similarly to (2) and (3), respectively. Note that traveling costs along the same geographical paths may vary for different k due to different crop-dependent operating machinery. The (expected) revenue from growing and marketing of crop $k \in \mathcal{K}$ on field $l \in \mathcal{L}$ is denoted by r_l^k . We assume a fixed cost of γ incurred for every additional crop. Maintenance cost per depot are given by n^d , $\forall d \in \mathcal{D}$. All costs shall be in monetary units.

Let us discuss decision variables. We distinguish between two major classes: *natural* and *auxiliary* decision variables. The first class comprises binary $x_{ij}^k \in \{0, 1\}$, $\forall i, j \in \mathcal{V}$, $\forall k \in \mathcal{K}$ with $x_{ij}^k = 1$ indicating arc (i, j) to be element of the optimal route for crop k . Symmetries are exploited whenever possible, i.e., x_{ji}^k is dismissed whenever $x_{ij}^k = x_{ji}^k$. For the symmetric case, we also assign $x_{dj}^k \in \{0, 1, 2\}$, $\forall d \in \mathcal{D}$, $\forall j \in \mathcal{L}$, $\forall k \in \mathcal{K}$, thereby indicating a visit of only field j for route corresponding to crop k . Further, there are binary $\delta_l^k \in \{0, 1\}$, $\forall l \in \mathcal{L}$, $\forall k \in \mathcal{K}$, with $\delta_l^k = 1$ indicating that crop k is assigned to field l . Integer m is such that $1 \leq m \leq K$ indicates the number of active crops in the optimal solution. As will be shown, *auxiliary* decision variables result from incorporating *logical* constraints into integer problems.

III. PROBLEM APPROACH

A. Framework and Approach

Assumption 1. Throughout a year, different crops have different, typically non-overlapping, harvesting times.

Assumption 2. *Throughout the harvest of any crop, harvesters are usually refueled and maintained on fields (i.e., there is no daily return to depots).*

Based on Assumptions 1 and 2, we approach Problem 1 using a mTSP-framework [4]. A route for each crop (crop-tour) and the fields correspond to a traveling salesman route and cities to be visited, respectively. Eventhough m routes (one for m crops) are planned simultaneously, the *sequential* harvesting times ultimately permit the framework. The mTSP-problem requires modification to combine with crop assignment and accounting for more constraints. To name just one example, a state space extension is required, i.e., adding the crop-dimension k to obtain x_{ij}^k instead of x_{ij} used in basic mTSP-formulations. We employ a integer programming (IP) framework for its ability to incorporate various constraints.

B. Clustering

A useful tool for us is grouping or *clustering* of fields, e.g., via the \tilde{k} -means algorithm [16]. As will be discussed below, it enables to upscale the number of fields that can be handled in a structured manner coupling crop assignment and route planning.

C. Pure Assignment Problems

The most basic IP for pure crop assignment to fields (without accounting for routing) is:

$$\min - \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} r_l^k \delta_l^k \quad (4a)$$

$$\text{s.t.} \quad \sum_{k \in \mathcal{K}} \delta_l^k = 1, \quad \forall l \in \mathcal{L}, \quad (4b)$$

$$\delta_l^k \in \{0, 1\}, \quad \forall l \in \mathcal{L}, \quad \forall k \in \mathcal{K}. \quad (4c)$$

Under additional assumptions $r_l^k = r^k$, $\forall l$ and $r^k \neq r^j$, $\forall k \neq j$, its optimal solution always assigns the most profitable crop (with largest r^k) to *all* fields. Let us discuss types of constraints that can be added.

First, we mention hard equality constraints motivated, for example, by *crop rotation* [3] or soil considerations (specific soils only admit specific crops),

$$\delta_l^k = 0, \quad \forall (l, k) \in \mathcal{R}, \quad (5)$$

where \mathcal{R} denotes a set of prohibited field/crop-combinations. Throughout, we assume that $\mathcal{R} = \{(l, k) : \sum_{k \in \mathcal{K}} \delta_l^k > 0, \forall l \in \mathcal{L}\}$, i.e., for every field there is at least one crop always admissible.

Second, we mention *diversification* inequality constraints

$$\sum_{l \in \mathcal{L}} g_l^k \delta_l^k \leq G^k, \quad \forall k = 0, \dots, K-2 \quad (6)$$

with $g_l^k \geq 0$ denoting weights (for example the ha-coverage or required production means for field l and crop k) and $G^k \geq 0$ the corresponding crop-related bounds, thereby *diversifying* crop-growth. Note that one crop $k = K-1$ was left unconstrained for feasibility. In general, when combining *both* aforementioned hard and inequality constraints without additional precaution, feasibility of the resulting IP cannot be guaranteed. Infeasibility results if these constraints enforce $\sum_{k \in \mathcal{K}} \delta_l^k = 0$, thereby violating (4b).

When including both crop rotation and diversification constraints, replacing (4a) by the relaxation $\sum_{k \in \mathcal{K}} \delta_l^k \leq 1$ always guarantees feasibility of (4). This is since these constraints can always be satisfied by $\delta_l^k = 0$.

Proposition 1. *The solution of the LP-relaxation of IP (4), and also including crop rotation constraints (5), is integer feasible, and thus solves these problems as well.*

Proof. We can easily summarize the IP as $\min\{c^T x : Ax = b, x_l \in \{0, 1\}, \forall l = 0, \dots, KL-1\}$. Its LP-relaxation reads $\min\{c^T x : Ax = b, x \geq 0\}$. By [17], if \tilde{A} is *totally unimodular*, the LP $\min\{c^T \tilde{x} : \tilde{A}\tilde{x} = \tilde{b}, \tilde{x} \in \mathbb{R}_+^n\}$ has an integral optimal solution for all integer vectors \tilde{b} for which it has a finite optimal value. It thus remains to show that A associated with the LP-relaxation of our IP is totally unimodular. By [18], a matrix A is totally unimodular if: (i) each entry is 0, 1 or -1 ; (ii) each column contains at most two non-zeros; (iii) the set \mathcal{N} or row indices of A can be partitioned into $\mathcal{N}_1 \cup \mathcal{N}_2$ such that in each column l with two non-zeros we have $\sum_{m_1 \in \mathcal{N}_1} a_{m_1 l} = \sum_{m_2 \in \mathcal{N}_2} a_{m_2 l}$. Condition (i) is trivially true from (4b) and (5). Regarding (ii), (4b) implies exactly one nonzero coefficient equal to 1 per column; to which, by (5), at most one more nonzero coefficient equal to 1 is added. For (iii), we partition sets \mathcal{N}_1 and \mathcal{N}_2 according to constraints (4b) and (5). Then $\sum_{m_1 \in \mathcal{N}_1} a_{m_1 l} = 1$ and $\sum_{m_2 \in \mathcal{N}_2} a_{m_2 l} = 1$ using the previous argument for (ii). This concludes the proof. \square

The consequence of Proposition 1 is that very large instances (with many fields and crops) of (4) with (5) can easily be solved. This is since there exist very efficient linear programming solvers. As a remark, the aforementioned inequality relaxation of (5) does not affect the totally unimodular property. This is since slack variables s_l can be introduced such that $\sum_{k \in \mathcal{K}} \delta_l^k + s_l = 1$, $s_l \in \{0, 1\}$, $\forall l \in \mathcal{L}$. They are not affecting (6), and thus a similar corresponding proposition and proof can be formulated. In contrast, adding diversification constraints (6), in general, render the LP-relaxation to not be integer feasible anymore.

D. TSP with Different Start and End Node

A useful tool for us is the TSP with different start and end node. As will be outlined below, it is employed for the routing within clusters of fields planting the same crop. Therefore, dropping superscript k , the IP is:

$$\min \sum_{i < j} c_{ij} x_{ij} \quad (7a)$$

$$\text{s.t.} \quad \sum_{j=1}^{N-2} x_{0j} = 1, \quad \sum_{j=1}^{N-2} x_{jN-1} = 1, \quad (7b)$$

$$\sum_{i < j} x_{il} + \sum_{l < j} x_{lj} = 2, \quad l = 1, \dots, N-2, \quad (7c)$$

$$\sum_{i < j; i, j \in S} x_{ij} \leq |S| - 1, \quad 3 \leq |S| \leq N-3, \quad \forall S \subseteq \mathcal{V} \setminus \{0, N-1\}, \quad (7d)$$

$$x_{ij} \in \{0, 1\}, \quad 0 \leq i < j, \quad j = 1, \dots, N-1, \quad (7e)$$

whereby we here set node 0 and $N-1$ as start and end node among a cluster of N nodes. Constraints (7b) and (7c) indicate that start and end node are incident to one edge, and all other

nodes incident to two, respectively. Under the assumption of symmetric edge weights, the *subtour elimination constraints* (SECs) [19] are given by (7d).

E. A Remark to Incorporating SECs in Integer Programs

Formulation (7) as well as the IPs following in Section IV include an exponential number of SECs [19]. We approach SECs in form of *separation algorithms* [20], i.e., by adding SECs sequentially as they are needed. With regard of (7), we start by solving it without (7d). If the result does not return any subtour, we have found the optimal solution. Otherwise, all detected subtours are added to (7) as SECs, and the IP is solved again. This is repeated until a solution without subtours is found (the optimal solution), or a maximal number of SEC-iterations is reached.

F. Harvesters Traveling as a Group

Remark 1. Assume that all of multiple harvesters are initially located at one depot to which they must return after processing all fields associated with a crop. An optimal policy is that all harvesters cover the fields together as a group, i.e., without distributing harvesters among different fields of the same crop.

Proof. Harvesters are not constrained by each other. They can always work in parallel on each field. The asymmetric case with fields ripening at different times already implies a unique optimal working sequence. For the symmetric case, an optimal route includes exactly two edges incident to the depot vertex. By symmetry it is thus always an optimal solution that all harvesters travel as a group. Any other initial distribution of harvesters to fields not connected to the depot vertex along the two aforementioned edges is already suboptimal by nonnegativity of traveling costs. \square

Multiple harvesters traveling as a group according to Remark 1 bears more practical advantages. In general, SUs must ideally be operated such that PUs (harvesters) can operate continuously to avoid any waiting times due to absent SUs for unloading. The rate at which harvesters are filled is not perfectly predictable due to varying crop returns even within one field. Having concentrated all SUs to one field is beneficial for robustness in that *multiple* harvesters can be served (instead of specific SU-PU couples) according to short-term freed capacities. Another advantage of allocating full concentration field-wise is facilitated supervision by the farm-manager.

Remark 1 has further implications for the general setting of multiple harvesters initially being distributed at multiple depots. Namely, optimization can be conducted *depot-wise* (instead of single harvester-wise). Besides this, no general a priori remark about routing of the harvester groups (depot groups) can be made. One heuristic strategy is that all harvesters assemble at the first field (of a crop-route) and then proceed group-wise. Such method is motivated by the fact that a timely agreement on harvest-start (e.g., a day ahead) permit all depot-groups to plan the travel in time and consequently start field and route coverage coordinately. In general, a distributed assignment of depot groups to fields is optimal. Consider, for example, depots far distant apart with fields clustered around each depot.

G. Logical Constraints

For the formulation of optimization problems, three classes of logical constraints are of particular interest. They are translated into integer linear inequalities using [21]. Let $\epsilon > 0$ be a small number (e.g., the machine precision), $b, b_1, b_2, b_3 \in \{0, 1\}$, $y \in \mathbb{R}$, and $f(x)$ such that $f : \mathbb{R}^{n_x} \rightarrow \mathbb{R}$ is linear, n_x the variable dimension, $f^{\max} = \max_{x \in \mathcal{X}} f(x)$ and $f^{\min} = \min_{x \in \mathcal{X}} f(x)$, where \mathcal{X} is a given bounded set.

- 1) The statement “ $b = 1$ if and only if $f(x) \leq 0$ and $b = 0$ otherwise” is equivalent to

$$f(x) \leq f^{\max}(1 - b), \quad f(x) \geq \epsilon + (f^{\min} - \epsilon)b. \quad (8)$$

- 2) The statement “ $b_3 = 1$ if and only if $b_1 = 1$ and $b_2 = 1$, and $b_3 = 0$ otherwise” is equivalent to $b_3 = b_1 b_2$ and is equivalent to

$$b_1 + b_2 - b_3 \leq 1, \quad b_3 \leq b_1, \quad b_3 \leq b_2. \quad (9)$$

- 3) The statement “ $y = f(x)$ if $b = 1$ and $y = 0$ otherwise” is equivalent to $y = b f(x)$ and is equivalent to

$$\begin{aligned} y &\leq f^{\max}b, \quad y \geq f^{\min}b, \quad y \leq f(x) - f^{\min}(1 - b), \\ y &\geq f(x) - f^{\max}(1 - b). \end{aligned} \quad (10)$$

H. Priority Constraints

To account for *a priori* experience about different sequences in ripeness of fields, *priority constraints* can be formulated. For example, relating to uncertainties, the sequence in which fields of the same crop ripe may vary, e.g., due to hillsides and varying soil. W.l.o.g., consider a statement such as “if fields a , b , and c are among the ones assigned to crop k , then the corresponding sequence for harvest shall be in order c , a and b ”. This can be modeled as $x_{ca}^k = \delta_c^k \delta_a^k$ and $x_{ab}^k = \delta_a^k \delta_b^k$ and can therefore be translated to *linear* integer inequalities by means of (9). Thus a sequential node-by-node procedure (first vertices (c, a) , then (a, b) , etc.) is recommended. Note that an *asymmetric* edge model has to be employed for all connections between vertices for which priorities are defined. For above example, we require $x_{ca}^k \neq x_{ac}^k$, since otherwise there is no *direction* information.

IV. PROBLEM SOLUTION

A. Integer Linear Programming

We propose eight integer linear programs, denoted by IP- n , $n = 1, \dots, 8$. IP- n and IP- $n + 4$ for $n = 1, \dots, 4$ are identical except that for the latter four, all K crops are enforced to be included in the solution, whereas the former four are formulated to also permit only any subset of K crops. This distinction has significant influence on computational complexity and problem formulation. Throughout this section, we use indices according to $d \in \mathcal{D}$, $i, j, l \in \mathcal{L}$ and $k \in \mathcal{K}$. Because of Assumption 1, we order crops in \mathcal{K} such that a low index indicating an earlier harvesting time. Throughout, SECs are handled as outlined in Section III-E.

IP-1. Let there be one depot $d \in \mathcal{D}$ from which all harvesters start and to which all harvesters return after each crop-route. According Remark 1, all harvesters are dispatched as a group. We propose the following IP:

$$\min \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{L}} c_{dj}^k x_{dj}^k + \sum_{k \in \mathcal{K}} \sum_{i < j} c_{ij}^k x_{ij}^k - \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} r_l^k \delta_l^k + \gamma m + n^d \eta^d \quad (11a)$$

$$\text{s.t. } x_{dl}^k + \sum_{i < l} x_{il}^k + \sum_{l < j} x_{lj}^k = 2\delta_l^k, \forall l \in \mathcal{L}, \forall k \in \mathcal{K}, \quad (11b)$$

$$\sum_{k \in \mathcal{K}} \delta_l^k = 1, \forall l \in \mathcal{L}, \quad (11c)$$

$$\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{L}} x_{dj}^k = 2m, \quad (11d)$$

$$\sum_{i < j; i, j \in S^k} x_{ij}^k \leq |S^k| - 1, \quad 3 \leq |S^k| \leq N - 1, \quad \forall k \in \mathcal{K}, S^k \subseteq \mathcal{V} \setminus \{d\}, \quad (11e)$$

$$x_{dj}^k \in \{0, 1, 2\}, \forall j \in \mathcal{L}, \forall k \in \mathcal{K}, \quad (11f)$$

$$x_{ij}^k \in \{0, 1\}, \quad 0 \leq i < j, \forall k \in \mathcal{K}, \quad (11g)$$

$$\delta_l^k \in \{0, 1\}, \forall l \in \mathcal{L}, \forall k \in \mathcal{K}, \quad (11h)$$

$$1 \leq m \leq K, \quad (11i)$$

with decision variables $x_{dj}^k, x_{ij}^k, \delta_l^k, m$ and assuming symmetric edge costs. In case of asymmetry regarding the traversal to and from depot vertices, we can replace $\sum_{d \in \mathcal{D}} x_{dl}^k$ and $x_{dl}^k \in \{0, 1, 2\}$ with $\sum_{d \in \mathcal{D}} x_{dl}^k + \sum_{d \in \mathcal{D}} x_{ld}^k$ and $x_{dl}^k, x_{ld}^k \in \{0, 1\}$, and similarly adopt (11d). *Diversification, crop rotation* and *priority* constraints discussed in Section III-C and III-H can easily be added. Similarly, constraints on the total traveling cost per crop can be formulated, for example, on *traveling time* with

$$\sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{L}} h_{dj}^k x_{dj}^k + \sum_{i < j} h_{ij}^k x_{ij}^k \leq T_{\text{win}}^k - \sum_{l \in \mathcal{L}} T_l^{\text{harv}, k} \delta_l^k, \quad (12)$$

for all $k \in \mathcal{K}$, where, for generality, we assumed the multi-depot case, and where h_{dj}^k and h_{ij}^k denote travel time along corresponding edges, T_{win}^k the harvesting window for crop k (typically multiple days), and $T_l^{\text{harv}, k}$ the required harvesting time (typically proportional to number of active harvesters) per field l and crop k . For a large number of fields, (12) become crucial because of limited harvesting time windows. In fact, in combination with the *partial service* constraints outlined in Seciton V-A they are central to limiting the maximum number of fields that should be serviced to still add monetary value. Note that without additional precaution *or* the relaxed service constraints to be discussed in Section V-A, (12) may invoke infeasible IPs. IP (11) has $n_z = KL + K \sum_{q=0}^{L-2} L - 1 - q + KL + 1$ integer decision variables. Term $n^d \eta^d$ in (11a) is constant since one depot is considered only. Constraints (11b) in combination with (11c) indicate that every field l is assigned exactly one crop k , and every field is incident to exactly two edges. Thus, these type of constraints couple crop assignment with vehicle routing. Constraint (11d) enforces the depot node to have exactly m edges incident, where m is a decision variable according to (11i). Under the assumption of symmetric edge weights, the SECs are given by (11e). By construction, any subtour is associated with exactly one crop $k \in \mathcal{K}$.

IP-2. Harvesters start the first crop-route from multiple depots. After *each* crop-route they must return to their original start depots. For IP-2, and under the assumption of symmetric

traveling costs, we model $c_{dj}^k = c_{dj}^{k, k_{\min}}, \forall k \in \mathcal{K}$. The remainder of IP-2 is identical to (11).

IP-3. Among a group of multiple available depots we seek the optimal selection, assuming that after every crop-route all harvesters will consequently start from and return to it. The following IP is proposed:

$$\min \sum_{d \in \mathcal{D}} \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{L}} c_{dj}^k x_{dj}^k + \sum_{k \in \mathcal{K}} \sum_{i < j} c_{ij}^k x_{ij}^k - \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} r_l^k \delta_l^k + \gamma m + \sum_{d \in \mathcal{D}} n^d \eta^d \quad (13a)$$

$$\text{s.t. } \sum_{d \in \mathcal{D}} x_{dl}^k + \sum_{i < l} x_{il}^k + \sum_{l < j} x_{lj}^k = 2\delta_l^k, \begin{cases} \forall l \in \mathcal{L}, \\ \forall k \in \mathcal{K}, \end{cases} \quad (13b)$$

$$\sum_{k \in \mathcal{K}} \delta_l^k = 1, \forall l \in \mathcal{L}, \quad (13c)$$

$$\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{L}} x_{dj}^k = 2p^d, \forall d \in \mathcal{D}, \quad (13d)$$

$$\sum_{d \in \mathcal{D}} \eta^d = 1, \quad (13e)$$

$$\eta^d \leq p^d \leq K\eta^d, \forall d \in \mathcal{D}, \quad (13f)$$

$$p^d \leq m - (1 - \eta^d), \forall d \in \mathcal{D}, \quad (13g)$$

$$p^d \geq m - K(1 - \eta^d), \forall d \in \mathcal{D}, \quad (13h)$$

$$\sum_{i < j; i, j \in S^k} x_{ij}^k \leq |S^k| - 1, \quad 3 \leq |S^k| \leq N - 1, \quad \forall k \in \mathcal{K}, S^k \subseteq \mathcal{V} \setminus \{d\}, \forall d \in \mathcal{D}, \quad (13i)$$

$$x_{dj}^k \in \{0, 1, 2\}, \forall d \in \mathcal{D}, \forall j \in \mathcal{L}, \forall k \in \mathcal{K}, \quad (13j)$$

$$x_{ij}^k \in \{0, 1\}, \quad 0 \leq i < j, \forall k \in \mathcal{K}, \quad (13k)$$

$$\delta_l^k \in \{0, 1\}, \forall l \in \mathcal{L}, \forall k \in \mathcal{K}, \quad (13l)$$

$$1 \leq m \leq K, \quad (13m)$$

$$\eta^d \in \{0, 1\}, \forall d \in \mathcal{D}, \quad (13n)$$

$$p^d \in \{0, 1, \dots, K\}, \forall d \in \mathcal{D}, \quad (13o)$$

with decision variables $x_{dj}^k, x_{ij}^k, \delta_l^k, m, \eta^d$ and p^d . IP (13) has $N_z = KL + K \sum_{q=0}^{L-2} L - 1 - q + KL + 1 + 2D$ integer decision variables. We discuss the key distinction w.r.t. IP-1. Since $D > 1$ is assumed, we model the decision to start from one of the depots as equality constraints

$$\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{L}} x_{dj}^k = 2m\eta^d, \forall d \in \mathcal{D}, \quad (14)$$

with $x_{dj}^k \in \{0, 1, 2\}, \eta^d \in \{0, 1\}, 1 \leq m \leq K$ and $\sum_{d \in \mathcal{D}} \eta^d = 1$. Since (14) is nonlinear, we introduce auxiliary variable $p^d = m\eta^d, \forall d \in \mathcal{D}$. By (10), this can be translated to linear inequality constraints (13f), (13g) and (13h). The cost coefficients c_{dj}^k are according to (2).

IP-4. Harvesters start the first crop-route from multiple depots. Then, they assemble at the first field for that route and consequently travel as one group until the last field to be covered for the last crop-route. With the exception of the last crop-route, all harvesters return to one depot which is selected optimally among the group of available depots. After the last field of the last crop-tour, all harvesters return to their original start depots. We therefore propose:

$$\begin{aligned} \min \quad & \sum_{d \in \mathcal{D}} \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{L}} c_{dj}^{k, k^{\min}} v_{dj}^k + c_{dj}^k x_{dj}^k - c_{dj}^k v_{dj}^k + \\ & \sum_{d \in \mathcal{D}} \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{L}} c_{jd}^{k, k^{\max}} w_{jd}^k + c_{jd}^k x_{jd}^k - c_{jd}^k w_{jd}^k \\ & \sum_{k \in \mathcal{K}} \sum_{i < j} c_{ij}^k x_{ij}^k - \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} r_l^k \delta_l^k + \gamma m + \sum_{d \in \mathcal{D}} n^d \eta^d \end{aligned} \quad (15a)$$

$$\text{s.t.} \quad \sum_{d \in \mathcal{D}} x_{dl}^k + \sum_{i < l} x_{il}^k + \sum_{l < j} x_{lj}^k + \sum_{d \in \mathcal{D}} x_{ld}^k = 2\delta_l^k, \begin{cases} \forall l \in \mathcal{L}, \\ \forall k \in \mathcal{K}, \end{cases} \quad (15b)$$

$$\sum_{k \in \mathcal{K}} \delta_l^k = 1, \forall l \in \mathcal{L}, \quad (15c)$$

$$\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{L}} x_{dj}^k + x_{jd}^k = 2p^d, \forall d \in \mathcal{D}, \quad (15d)$$

$$\sum_{d \in \mathcal{D}} \eta^d = 1, \sum_{k \in \mathcal{K}} \tilde{\alpha}^k = 1, \sum_{k \in \mathcal{K}} \tilde{\beta}^k = 1, \quad (15e)$$

$$\tilde{\alpha}^0 = \alpha^0, \tilde{\beta}^{K-1} = \alpha^{K-1}, \quad (15f)$$

$$\sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{L}} \sum_{k \in \mathcal{K}} v_{dj}^k = 1, \sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{L}} \sum_{k \in \mathcal{K}} w_{jd}^k = 1 \quad (15g)$$

$$\sum_{j \in \mathcal{L}} x_{dj}^k = \eta^d, \forall d \in \mathcal{D}, \forall k \in \mathcal{K}, \quad (15h)$$

$$\sum_{j \in \mathcal{L}} x_{jd}^k = \eta^d, \forall d \in \mathcal{D}, \forall k \in \mathcal{K}, \quad (15i)$$

$$\eta^d \leq p^d \leq K, \forall d \in \mathcal{D}, \quad (15j)$$

$$p^d \leq m - (1 - \eta^d), \forall d \in \mathcal{D}, \quad (15k)$$

$$p^d \geq m - K(1 - \eta^d), \forall d \in \mathcal{D}, \quad (15l)$$

$$1 - \sum_{l \in \mathcal{L}} \delta_l^k \leq 1 - \alpha^k, \forall k \in \mathcal{K}, \quad (15m)$$

$$1 - \sum_{l \in \mathcal{L}} \delta_l^k \geq \epsilon + (-L + 1 - \epsilon)\alpha^k, \forall k \in \mathcal{K}, \quad (15n)$$

$$\alpha^k + (1 - \sum_{\tau=0}^{k-1} \tilde{\alpha}^\tau) - \tilde{\alpha}^k \leq 1, \forall k = 1, \dots, K-1, \quad (15o)$$

$$\tilde{\alpha}^k \leq \alpha^k, \tilde{\alpha}^k \leq 1 - \sum_{\tau=0}^{k-1} \tilde{\alpha}^\tau, \forall k = 1, \dots, K-1. \quad (15p)$$

$$\alpha^{K-2-k} + (1 - \sum_{\tau=1}^{1+k} \tilde{\beta}^{K-2-k+\tau}) - \tilde{\beta}^{K-2-k} \leq 1, \quad \forall k = 0, \dots, K-2, \quad (15q)$$

$$\tilde{\beta}^{K-2-k} \leq \alpha^{K-2-k}, \forall k = 0, \dots, K-2, \quad (15r)$$

$$\tilde{\beta}^{K-2-k} \leq 1 - \sum_{\tau=1}^{1+k} \tilde{\beta}^{K-2-k+\tau}, \forall k = 0, \dots, K-2, \quad (15s)$$

$$\tilde{\alpha}^k + x_{dj}^k - v_{dj}^k \leq 1, v_{dj}^k \leq \tilde{\alpha}^k, v_{dj}^k \leq x_{dj}^k, \quad \forall d \in \mathcal{D}, \forall j \in \mathcal{L}, \forall k \in \mathcal{K}, \quad (15t)$$

$$\tilde{\beta}^k + x_{jd}^k - w_{jd}^k \leq 1, w_{jd}^k \leq \tilde{\beta}^k, w_{jd}^k \leq x_{jd}^k, \quad \forall d \in \mathcal{D}, \forall j \in \mathcal{L}, \forall k \in \mathcal{K}, \quad (15u)$$

$$\sum_{i < j; i, j \in S^k} x_{ij}^k \leq |S^k| - 1, 3 \leq |S^k| \leq N - 1, \quad \forall k \in \mathcal{K}, S^k \subseteq \mathcal{V} \setminus \{d\}, \quad (15v)$$

with decision variables

$$x_{dj}^k \in \{0, 1\}, \forall d \in \mathcal{D}, \forall j \in \mathcal{L}, \forall k \in \mathcal{K}, \quad (16a)$$

$$x_{ij}^k \in \{0, 1\}, 0 \leq i < j, \forall k \in \mathcal{K}, \quad (16b)$$

$$\delta_l^k \in \{0, 1\}, \forall l \in \mathcal{L}, \forall k \in \mathcal{K}, \quad (16c)$$

$$1 \leq m \leq K, \quad (16d)$$

$$\eta^d \in \{0, 1\}, \forall d \in \mathcal{D}, \quad (16e)$$

$$p^d \in \{0, 1, \dots, K\}, \forall d \in \mathcal{D}, \quad (16f)$$

$$\alpha^k, \tilde{\alpha}^k, \tilde{\beta}^k \in \{0, 1\}, \forall k \in \mathcal{K}, \quad (16g)$$

$$v_{dj}^k, w_{jd}^k \in \{0, 1\}, \forall d \in \mathcal{D}, \forall j \in \mathcal{L}, \forall k \in \mathcal{K}, \quad (16h)$$

$$x_{jd}^k \in \{0, 1\}, \forall d \in \mathcal{D}, \forall j \in \mathcal{L}, \forall k \in \mathcal{K}. \quad (16i)$$

For the formulation of crop- and depot-dependent cost coefficients, the minimum and maximum active crop-indices need to be identified. Let therefore $\alpha^k \in \{0, 1\}$ indicate if crop k is active in the sense of $\alpha^k = 1$ if $\sum_{l \in \mathcal{L}} \delta_l^k \geq 1$. By (8), this translates to (15m) and (15n). We introduced auxiliary variables $\tilde{\alpha}^k, \tilde{\beta}^k \in \{0, 1\}$ indicating if crop k is the *smallest*- or *largest*-indexed active crop, respectively ($k = k^{\min}$ and $k = k^{\max}$). It holds that $\sum_{k \in \mathcal{K}} \tilde{\alpha}^k = 1$ and $\sum_{k \in \mathcal{K}} \tilde{\beta}^k = 1$. We then derive the nonlinear relations $\tilde{\alpha}^0 = \alpha^0, \tilde{\alpha}^1 = \alpha^1(1 - \tilde{\alpha}^0), \tilde{\alpha}^2 = \alpha^2(1 - \tilde{\alpha}^1 - \tilde{\alpha}^0), \dots$, which can be translated to

$$\tilde{\alpha}^0 = \alpha^0, \quad (17)$$

$$\alpha^k + (1 - \sum_{\tau=0}^{k-1} \tilde{\alpha}^\tau) - \tilde{\alpha}^k \leq 1, \forall k = 1, \dots, K-1, \quad (18)$$

$$\tilde{\alpha}^k \leq \alpha^k, \tilde{\alpha}^k \leq 1 - \sum_{\tau=0}^{k-1} \tilde{\alpha}^\tau, \forall k = 1, \dots, K-1. \quad (19)$$

Similarly, starting the iteration from highest $k = K-1$ with $\tilde{\beta}^{K-1} = \alpha^{K-1}$, we can derive nonlinear relations for $\tilde{\beta}^k$ to ultimately obtain (15q), (15r) and (15s). Suppose the *path-dependent* part of the cost function taking the *nonlinear* form $\sum_{d \in \mathcal{D}} \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{L}} (c_{dj}^{k, k^{\min}} \tilde{\alpha}^k + c_{dj}^k (1 - \tilde{\alpha}^k)) x_{dj}^k + \sum_{d \in \mathcal{D}} \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{L}} (c_{jd}^{k, k^{\max}} \tilde{\beta}^k + c_{jd}^k (1 - \tilde{\beta}^k)) x_{jd}^k$ with $c_{dj}^{k, k^{\min}} \geq 0$ and $c_{jd}^{k, k^{\max}} \geq 0$ denoting cost-coefficients that are distinct for the first (i.e., $k = k^{\min}$ or $\tilde{\alpha}^k = 1$) and last (i.e., $k = k^{\max}$ or $\tilde{\beta}^k = 1$) crop-route. Then, auxiliary variables $v_{dj}^k \in \{0, 1\}$ and $w_{jd}^k \in \{0, 1\}$ need to be introduced with $\sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{L}} \sum_{k \in \mathcal{K}} v_{dj}^k = 1$ and $\sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{L}} \sum_{k \in \mathcal{K}} w_{jd}^k = 1$. They are related according $v_{dj}^k = \tilde{\alpha}^k x_{dj}^k$ and $w_{jd}^k = \tilde{\beta}^k x_{jd}^k, \forall d \in \mathcal{D}, j \in \mathcal{L}, k \in \mathcal{K}$ and can be translated to integer linear inequalities according to (9). The objective function part above can now be expressed *linearly* dependent on decision variables, see (15a).

IP-5. We fix $m = K$. Thus, there is one decision variable less w.r.t. IP-1. IP-5 is identical to (11) with few exceptions: γm in (11a) is constant, (11i) can be omitted, and constraints (11d) are replaced by

$$\sum_{j \in \mathcal{L}} x_{dj}^k = 2, \forall k \in \mathcal{K}.$$

IP-6. We can adopt IP-5 with the exception of modified cost coefficients according to $c_{dj}^k = c_{dj}^{k, k^{\min}}, \forall k \in \mathcal{K}$.

IP-7. We fix $m = K$ in IP-3. As a consequence, (14) is rendered linear and (13) simplifies to

$$\min \sum_{d \in \mathcal{D}} \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{L}} c_{dj}^k x_{dj}^k + \sum_{k \in \mathcal{K}} \sum_{i < j} c_{ij}^k x_{ij}^k - \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} r_l^k \delta_l^k + \gamma K + \sum_{d \in \mathcal{D}} n^d \eta^d \quad (20a)$$

$$\text{s.t.} \quad \sum_{d \in \mathcal{D}} x_{dl}^k + \sum_{i < l} x_{il}^k + \sum_{l < j} x_{lj}^k = 2\delta_l^k, \begin{cases} \forall l \in \mathcal{L}, \\ \forall k \in \mathcal{K}, \end{cases} \quad (20b)$$

$$\sum_{k \in \mathcal{K}} \delta_l^k = 1, \quad \forall l \in \mathcal{L}, \quad (20c)$$

$$\sum_{j \in \mathcal{L}} x_{dj}^k = 2\eta^d, \quad \forall d \in \mathcal{D}, \quad \forall k \in \mathcal{K}, \quad (20d)$$

$$\sum_{d \in \mathcal{D}} \eta^d = 1, \quad (20e)$$

$$\sum_{i < j; i, j \in S^k} x_{ij}^k \leq |S^k| - 1, \quad 3 \leq |S^k| \leq N - 1, \quad \forall k \in \mathcal{K}, \quad S^k \subseteq \mathcal{V} \setminus \{d\}, \quad \forall d \in \mathcal{D}, \quad (20f)$$

$$x_{dj}^k \in \{0, 1, 2\}, \quad \forall d \in \mathcal{D}, \quad \forall j \in \mathcal{L}, \quad \forall k \in \mathcal{K}, \quad (20g)$$

$$x_{ij}^k \in \{0, 1\}, \quad 0 \leq i < j, \quad \forall k \in \mathcal{K}, \quad (20h)$$

$$\delta_l^k \in \{0, 1\}, \quad \forall l \in \mathcal{L}, \quad \forall k \in \mathcal{K}, \quad (20i)$$

$$\eta^d \in \{0, 1\}, \quad \forall d \in \mathcal{D}. \quad (20j)$$

IP-8. Constraining $m = K$ significantly simplifies (15) to

$$\min \sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{L}} c_{dj}^{0, k_{\min}} x_{dj}^0 + c_{jd}^{K-1, k_{\max}} x_{jd}^{K-1} + \gamma K + \sum_{d \in \mathcal{D}} n^d \eta^d + \sum_{d \in \mathcal{D}} \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{L}} c_{dj}^k x_{dj}^k + \sum_{k \in \mathcal{K}} \sum_{i < j} c_{ij}^k x_{ij}^k - \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} r_l^k \delta_l^k \quad (21a)$$

$$\text{s.t.} \quad \sum_{d \in \mathcal{D}} x_{dl}^k + \sum_{i < l} x_{il}^k + \sum_{l < j} x_{lj}^k + \sum_{d \in \mathcal{D}} x_{ld}^k = 2\delta_l^k, \begin{cases} \forall l \in \mathcal{L}, \\ \forall k \in \mathcal{K}, \end{cases} \quad (21b)$$

$$\sum_{k \in \mathcal{K}} \delta_l^k = 1, \quad \forall l \in \mathcal{L}, \quad (21c)$$

$$\sum_{j \in \mathcal{L}} x_{dj}^k + x_{jd}^k = 2\eta^d, \quad \forall d \in \mathcal{D}, \quad \forall k \in \mathcal{K}, \quad (21d)$$

$$\sum_{j \in \mathcal{L}} x_{dj}^k = \eta^d, \quad \sum_{j \in \mathcal{L}} x_{jd}^k = \eta^d, \quad \forall d \in \mathcal{D}, \quad \forall k \in \mathcal{K}, \quad (21e)$$

$$\sum_{d \in \mathcal{D}} \eta^d = 1, \quad (21f)$$

$$\sum_{i < j; i, j \in S^k} x_{ij}^k \leq |S^k| - 1, \quad 3 \leq |S^k| \leq N - 1, \quad \forall k \in \mathcal{K}, \quad S^k \subseteq \mathcal{V} \setminus \{d\}, \quad \forall d \in \mathcal{D}, \quad (21g)$$

$$x_{dj}^k \in \{0, 1\}, \quad \forall d \in \mathcal{D}, \quad \forall j \in \mathcal{L}, \quad \forall k \in \mathcal{K}, \quad (21h)$$

$$x_{ij}^k \in \{0, 1\}, \quad 0 \leq i < j, \quad \forall k \in \mathcal{K}, \quad (21i)$$

$$\delta_l^k \in \{0, 1\}, \quad \forall l \in \mathcal{L}, \quad \forall k \in \mathcal{K}, \quad (21j)$$

$$\eta^d \in \{0, 1\}, \quad \forall d \in \mathcal{D}, \quad (21k)$$

$$x_{jd}^k \in \{0, 1\}, \quad \forall d \in \mathcal{D}, \quad \forall j \in \mathcal{L}, \quad \forall k \in \mathcal{K}, \quad (21l)$$

with $\tilde{\mathcal{K}} = \mathcal{K} \setminus \{0, K - 1\}$.

B. Comparison of IP-formulations

Let us first compare above IP-formulations *coupling* crop assignment with routing vs. a two-stage approach with the first stage the solution of an assignment problem and the second stage the solution of one TSP for each crop. Any coupling

approach is *always* at least as good as any two-stage method. This is since the optimal solution of the latter is always a feasible solution of the former method. Without making further assumption, no further general statements can be made. Under assumptions, simple heuristic algorithms based on inequality checks can be developed and applied to the two-stage solution to determine suboptimality w.r.t. the coupling solution but without having solved the coupling IP.

Let us denote the objective functions of IP-2 and IP-3 by J_{IP-2} and J_{IP-3} , respectively.

Proposition 2. *It always holds that $J_{IP-3} \leq J_{IP-2}$.*

Proof. The proof is by contradiction. Let us assume $J_{IP-2} < J_{IP-3}$. J_{IP-2} and J_{IP-3} differ by cost coefficients $c_{dj}^k = c_{dj}^{k, k_{\min}}$, $\forall k \in \mathcal{K}$ and c_{dj}^k in (2), respectively. By linearity of J_{IP-2} and the definition of $c_{dj}^{k, k_{\min}}$ according to (3), and by nonnegativity of \tilde{c}_{dj}^k , J_{IP-2} can always be lowered by concentrating all harvesters, $\sum_{d \in \mathcal{D}} N_{\tilde{d}}^{\text{harv}, k}$, to the most cost-efficient depot. This is the IP-3 solution and therefore contradicts our assumption. The equality-part is because a special case of IP-2 is that none harvesters are initially located at any of the depots except the optimal one according to IP-3. This concludes the proof. \square

It always is $J_{IP-3} \leq J_{IP-1}$ since the latter single depot case is always included in the former multiple depot case.

Generalizing statements regarding J_{IP-1} vs. J_{IP-2} , and likewise for J_{IP-3} vs. J_{IP-4} cannot be made. This is because it is always possible to create counterexamples in favor of one or another solution.

It always is $J_{IP-n} \leq J_{IP-(n+4)}$, $\forall n = 1, \dots, 4$. This is because of the greater freedom in *not* having to use *all* crops for the final solution for the first four cases.

C. Main Algorithm

The main algorithm of this paper is summarized in Algorithm 1. See Figure 3 for its visualization. It is used for *crop assignment plus routing* (CApR). We denote the reversing of a list or sequence of elements by the $\text{flip}(\cdot)$ -operator. Several remarks can be made.

First, Algorithm 1 is motivated to handle a large number of fields. This is achieved by the proposed layered approach. It comes, however, at the cost of returning, in general, a suboptimal solution. The exact and global optimum solution is attained for $\tilde{k} = L$. In practice, it is recommended to increase the number of clusters as much as computational power and available IP-solver permit, ideally, until $\tilde{k} = L$ and Step 4 of Algorithm can be omitted entirely.

Second, the relations between various J_{IP-n} for $n = 1, \dots, 8$, as discussed in Sections IV-B, can in general *not* be translated to the corresponding objective values of the CApR- n solutions. This is because of the *heuristic* (layered) nature of Algorithm 1. For instance, in general, it does not always hold that $J_{CApR-3} \leq J_{CApR-2}$.

Third, under the absence of priority constraints according to Section III-H, there exist two directions in which to traverse any crop-tour. The traversal direction affects the closest fields between any pair of consecutive clusters. Consequently, the

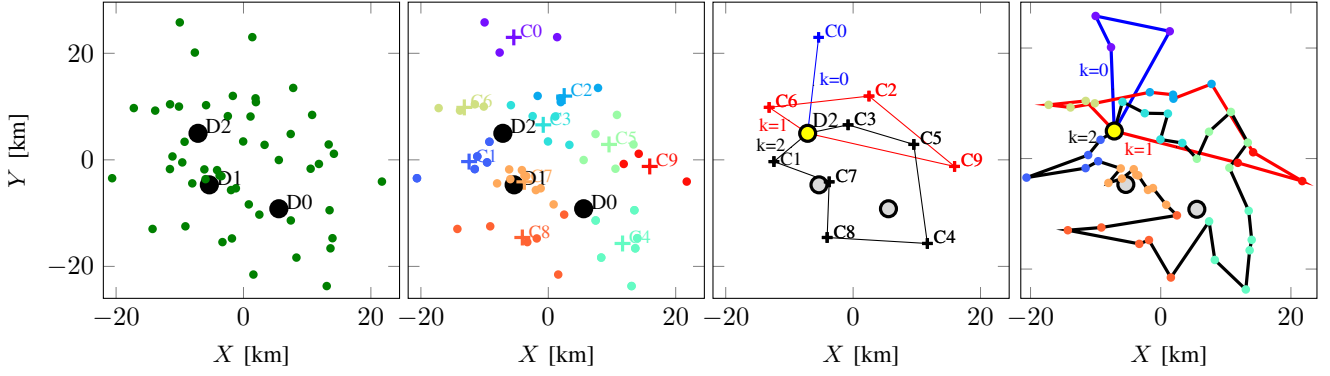


Fig. 3. Illustration of steps 1 to 4 (Plot 1 to 4 from left to right) of Algorithm 1. (Plot 1) Three depots (D0, D1 and D2) and 50 fields are visualized by the black and green balls, respectively. (Plot 2) The fields are assigned to $\tilde{k} = 10$ clusters (C0, ..., C9). All fields belonging to the same cluster are colored correspondingly. The cross-signs indicate the \tilde{k} centroids and are labeled accordingly. (Plot 3) Results of IP-7 applied to the \tilde{k} centroids. (Plot 4) Results of CAPR-7. For visualization, the fields are colored according to the clustering result. Labels $k = 0, k = 1$ and $k = 2$ indicate the *first* traversal of each crop-tour, whereby a *crop-tour* denotes the harvesting routes associated with a specific crop k .

Algorithm 1 CAPR- n

- 1: **Input:** $\{P_l\}_{l=0}^{L-1}$, $\{P_d\}_{d=0}^{D-1}$, c_{dj}^k , c_{ij}^k , $c_{dj}^{k, \min}$, $c_{jd}^{k, \max}$, c_{jd}^k , r_l^k , γ , $\{\eta^d\}_{d=0}^{D-1}$ and \tilde{k} according to Section II-B.
- 2: **Clustering:**
 - cluster L fields according to an arbitrary criterion, e.g., spatially based according to k -means [16].
 - let the sets of fields associated with each cluster be denoted by $\mathcal{L}_{z, \xi} \subset \mathcal{L}$, $\forall \xi = 0, \dots, \tilde{k} - 1$.
 - let the set of clusters be denoted by \mathcal{L}_z with $|\mathcal{L}_z| = \tilde{k}$.
 - assign a coordinate $P_{l_z} \in \mathbb{R}^2$, $\forall l_z \in \mathcal{L}_z$, to each cluster (the centroids for standard k -means [16]).
 - compute $c_{dj_z}^k$, $c_{i_z j_z}^k$, $c_{dj_z}^{k, \min}$, $c_{j_z d}^{k, \max}$, $c_{j_z d}^k$, $\forall i_z, j_z \in \mathcal{L}_z$, similarly to Section II-B.
 - compute $r_{l_z}^k = \sum_{l \in \mathcal{L}_{z, \xi}} r_l^k$, $\forall l_z \in \mathcal{L}_z$, $\xi = 0, \dots, \tilde{k} - 1$.
- 3: **Integer Programming (IP- n):**
 - solve IP- n from Section IV-A for the clustering result of Step 2, replacing \mathcal{L} by \mathcal{L}_z and cost coefficients accordingly.
 - let the resulting set of active crops and optimal basis depot be denoted by $\mathcal{M}^* \subseteq \mathcal{K}$ and $d^* \in \mathcal{D}$, respectively.
 - let \mathcal{C}^k denote the sequence of clusters $\forall k \in \mathcal{M}^*$, whereby every sequence starts and ends at $d^* \in \mathcal{D}$.
- 4: **From Cluster- to Field-sequences:**
 - FOR $k \in \mathcal{M}^*$:
 - define $\mathcal{C}^{k,1} = \mathcal{C}^k$ and $\mathcal{C}^{k,2} = \text{flip}(\mathcal{C}^k)$.
 - FOR $i = 1, 2$:
 - FOR $\mathcal{C}^{k,i}$:
 - find the closest fields between any pair of consecutive clusters $c^{(t)}, c^{(t+1)} \in \mathcal{C}^{k,i}$ within the $\mathcal{C}^{k,i}$ -tour, and where $t = 0, \dots, |\mathcal{C}^{k,i}|$.
 - let the two fields associated with each cluster $c^{(t)}$ be denoted by $s^{(t)}$ and $e^{(t)}$.
 - for each cluster $c^{(t)}$, $\forall t$, solve a TSP according to Section III-D connecting $s^{(t)}$ and $e^{(t)}$ to obtain a corresponding field-sequence $f^{(t)} = \{s^{(t)}, \dots, e^{(t)}\}$.
 - concatenate all field-sequences to crop-tour $\mathcal{F}^{k,i} = \{f^{(0)}, \dots, f^{(|\mathcal{C}^{k,i}|)}\}$ and determine its pathlength $d^{k,i}$.
 - IF $d^{k,1} < d^{k,2}$: $\mathcal{F}^{k,*} = \mathcal{F}^{k,1}$, else $\mathcal{F}^{k,*} = \mathcal{F}^{k,2}$.
 - 5: **Output:**
 - set of active crops \mathcal{M}^* and basis depot selection $d^* \in \mathcal{D}$.
 - crop assignment to every field, $\delta_l^{k,*}$, $\forall l \in \mathcal{L}, \forall k \in \mathcal{M}^*$.
 - crop-tour $\mathcal{F}^{k,*}$, $\forall k \in \mathcal{M}^*$.

Algorithm 2 Renting out and Taking Leases

- 1: Define all fields considered by $\mathcal{L} = \mathcal{L}^{\text{own}} \cup \mathcal{L}^{\text{ptl}}$.
- 2: Define the set of fields of interest by $\tilde{\mathcal{L}} = \mathcal{L}^{\text{pro}} \cup \mathcal{L}^{\text{ptl}}$.
- 3: Modeling according farmer's *own* production means.
 - determine parameters of Step 1 of Algorithm 1, $\forall l \in \mathcal{L}$.
- 4: **Solve** a *relaxed* CAPR- n for any desired $n = 1, \dots, 8$.
- 5: Determine $\mathcal{L}^{\text{ntl}} = \{l \in \mathcal{L}^{\text{ptl}} : \delta_l^k = 0, \forall k \in \mathcal{K}\}$.
 - *not* take a lease on any of these fields.
- 6: Determine $\mathcal{L}^{\text{ro}} = \{l \in \mathcal{L}^{\text{pro}} : \delta_l^k = 0, \forall k \in \mathcal{K}\}$.
 - rent out all of these fields (*any* positive return is good).
- 7: **Solve** standard CAPR- n for $\mathcal{L}^1 = \mathcal{L} \setminus \{\mathcal{L}^{\text{ntl}} \cup \mathcal{L}^{\text{ro}}\}$.
 - denote its objective value by $J_{\mathcal{L}^1}$.
- 8: **Solve** standard CAPR- n for \mathcal{L}^{own} .
 - denote its objective value by $J_{\mathcal{L}^{\text{own}}}$.
- 9: Take leases of fields $\mathcal{L}^{\text{ptl}} \setminus \mathcal{L}^{\text{ntl}}$ for the *overall* payment rate of at most $\Delta J = J_{\mathcal{L}^1} - J_{\mathcal{L}^{\text{own}}}$.

TSP-solution for each cluster, and thereby ultimately the total pathlength of the crop-tour, is affected, too. This motivated to test *both* cluster-sequences as indicated in Step 4. As stated, Algorithm 1 does not account for priority constraints, i.e., for a priori modeling of field ripeness sequences. Therefore, Step 2 and 4 require modification and clustering must be conducted according to an objective accounting for ripeness level. As a consequence, the traversal direction for Step 4 would also be fixed.

Fourth, the result of Algorithm 1 could in principle be further refined by heuristic local searches such as, for example, *hill climbing*, i.e., the local exchange of field-pairs within a crop-tour sequence if it improves the CAPR- n objective function value. Naturally, at this stage, local field sequences can also be exchanged manually by farm operators.

V. EXTENSIONS

A. Financial Considerations Regarding Leasing

The clustering step of Algorithm 1 does not necessarily have to be conducted according to spatial proximity of fields. Fields can be clustered arbitrarily. Also, single fields can be assigned to a single cluster for special analysis. For leasing considerations, the *partial* service of a subset of fields is of interest. Let

subset $\tilde{\mathcal{L}} \subseteq \mathcal{L}$ denote all fields for which we do not necessarily want to enforce field service but contemplate leasing options. Then, for IP-3, we maintain equality constraints (13b) and (13c) only for $\mathcal{L} \setminus \tilde{\mathcal{L}}$, and define relaxed inequalities

$$\sum_{d \in \mathcal{D}} x_{dl}^k + \sum_{i < l} x_{il}^k + \sum_{l < j} x_{lj}^k \leq 2\delta_l^k, \forall l \in \tilde{\mathcal{L}}, \forall k \in \mathcal{K}, \quad (22)$$

$$\sum_{k \in \mathcal{K}} \delta_l^k \leq 1, \forall l \in \tilde{\mathcal{L}}. \quad (23)$$

We similarly relax corresponding constraints for all other IP- n . Any CApR- n including such constraints, shall be denoted as *relaxed* CApR- n . In contrast, the original problem according to Section IV-C is referred to as standard CApR- n .

An important financial consideration for every farm is to decide upon *either* service *or* renting out of one's fields, and additionally the decision upon taking of leases on additional fields for coverage. Let us denote the sets of corresponding fields by \mathcal{L}^{own} (farmer's own fields), $\mathcal{L}^{\text{pro}} \subset \mathcal{L}^{\text{own}}$ (potential rent outs) and \mathcal{L}^{ptl} (potential fields for taking leases upon), respectively. Then, Algorithm 2 provides guidelines for decision making. Let us elaborate Step 4 of Algorithm 2. Suppose a field does not improve the total financial return, typically, because of too expensive production costs (consider, for example, fields very distant apart from depots) or constraints such as (12) for limited harvesting windows. Then, renting out is profitable, in theory, already for any positive return. In practice, the farmer is naturally advised to negotiate renting out rates as favorably as possible. Let us also discuss Step 9 of Algorithm 2. In contrast to pure assignment problems, the maximum leasing rate ΔJ cannot easily be distributed among corresponding fields. This is because monetary profits are nonlinearly related to crop returns because of the coupling with routing decisions. Importantly, the precise distribution of leasing rates of individual fields is not relevant as long as it *overall* does not surpass ΔJ . Thus, ΔJ provides the farmer with an *upper bound* on profitable leasing rates. If ΔJ cannot be attained in negotiations, different \mathcal{L}^{ptl} should be decided and Algorithm 2 solved again. This is repeated until a corresponding upper bound can be satisfied, or, ultimately, $\mathcal{L}^{\text{own}} \setminus \mathcal{L}^{\text{ro}}$ are serviced.

The second financial consideration is motivated by the comparison of objective values for CApR- n . It permits to determine "fair" prices for leasing when sheltering machinery at the various depots. It is envisioned that *all* collaborating farmers first involve in accurate system modeling (cost coefficients), before then solving either all of CApR- n , $\forall n = 1, \dots, 4$, or all of CApR- n , $\forall n = 5, \dots, 8$. Specifically, the difference in objective values between CApR-2 (or CApR-7 for enforcement of all K crops in the solution) and the remaining CApR- n then permits to determine an *upper bound* on leasing rates for depot usage.

B. Application in Practice

For operations planning in practice, detailed modeling of the parameters listed in Step 1 of Algorithm 1 is of paramount importance for optimal results. Historical field and crop yield data must serve as basis. By the selection of c_{ij}^k , computational complexity can be reduced by pruning specific undesired

field connections from a path network, thereby implicitly also influencing priority constraints. Large fields often have multiple possible field entrance and exit points. This may significantly affect travel distances between fields. In fact, field coverage patterns and in-field navigation [13], [22], [23] can also be co-planned a priori to account for crop-tours efficiently linking fields planting the same crops. This is subject of ongoing work.

VI. NUMERICAL SIMULATIONS

For the solution of integer programs, we employ the domain-specific language CVXPY for optimization embedded in Python [24] with default settings. All numerical experiments throughout this paper were conducted on a laptop running Ubuntu 16.04 equipped with an Intel Core i7 CPU @2.80GHz×8, 15.6GB of memory.

Problem data is generated randomly with realistic parameter settings from farming in Northern Germany. For the first numerical simulation experiments, we assume three depots, a maximal number of three crops and 50 fields, i.e., $D = 3$, $K = 3$ and $L = 50$. Fields are clustered spatially into sets according to step 2 of Algorithm 1, whereby we selected $\tilde{k} = 10$. Field and depot locations are generated randomly according to a Gaussian distribution centered at the origin with standard deviations $\sigma_d = 10\text{km}$, $\forall d \in \mathcal{D}$, and $\sigma_l = 15\text{km}$, $\forall l \in \mathcal{L}$. To each depot, we randomly assign a number of harvesters according $N_d^{\text{harv},k} = \max(1, \lfloor 5u_d \rfloor)$, $u_d \sim \mathcal{U}(0, 1)$, $\forall d \in \mathcal{D}$, where $\mathcal{U}(0, 1)$ denotes the Uniform distribution with zero mean and unit variance, and $\lfloor \cdot \rfloor$ denotes rounding to the next smallest integer. Normalized traveling costs per harvester and km are set as $\bar{c} = 30 \frac{\text{€}}{\text{km}}$. We assume a cost of $\gamma = 1000\text{€}$ for every planted crop. Here, maintenance costs are assumed to be identical for all depots. W.l.o.g., we therefore set $n^d = 0$, $\forall d \in \mathcal{D}$. Realistic normalized monetary returns in € per ha and crop are determined as mean values from intermediate soil qualities and crop yields in Northern Germany. They are summarized in Table II. Regarding the monetary return per field and crop, we considered two settings. First, we generate field sizes in hectares according to $s_l = \max(20 + 10u_d, 1)$, $u_d \sim \mathcal{N}(0, 1)$, $\forall l \in \mathcal{L}$, where $\mathcal{N}(0, 1)$ denotes the Gaussian distribution with zero mean and unit variance. In combination with $L = 50$ this results approximately in a total coverage size of 1000ha. According to survey [25], in *all* of Germany there are 299134 farming businesses of which only 1502 have a size of more than 1000ha. The field sizes are then multiplied with \tilde{r}^k according to Table II, to yield $r_l^k = s_l \tilde{r}^k$, $\forall l \in \mathcal{L}$, $k \in \mathcal{K}$. This method of data generation is intuitive. Since normalized monetary return is considerably higher for wheat than for barley and rapeseed, the application of Algorithm 1 typically assigns wheat to *all* fields, unless crop rotation constraints, or diversification constraints, as in CApR- n for $n = 5, \dots, 8$, are included. In the latter cases, the crop with smallest monetary return is assigned to the cluster with smallest field area, and the crop with second-smallest return to the second-smallest area and so forth. In a second setting, and to add more variety, we therefore generated monetary returns per field and crop according to

$$r_l^k = \max(20 + 10u_l^k, 1) \tilde{r}^k, \forall l \in \mathcal{L}, k \in \mathcal{K}, \quad (24)$$

TABLE I. Experiment 1. The percentage out of the 50 simulation experiments for which an IP- n solution could be found in less than 200 SEC-iterations is denoted by $P_{\text{conv}}^{\text{IP}-n}$. The absolute average monetary objective value is denoted by $\bar{J}^{\text{CApR}-n}$. The average number of decision variables, SEC-iterations, number of equality constraints, inequality constraints when first omitting SECs and for the final SEC-iteration (before convergence) are denoted by $N_z^{\text{IP}-n}$, $\bar{N}_{\text{iterSEC}}^{\text{IP}-n}$, $N_{\text{eq,NoSEC}}^{\text{IP}-n}$, $N_{\text{ineq,NoSEC}}^{\text{IP}-n}$ and $\bar{N}_{\text{ineq,finalIP}}^{\text{IP}-n}$, respectively. Average combined CPU-time for the solution of all SEC-iterations is $\bar{T}_{\text{CPU}}^{\text{IP}-n}$.

	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$	unit
$\bar{J}^{\text{CApR}-n}$	746766.5	743866.0	748922.1	745887.6	744816.7	743201.4	746945.8	745970.1	€
$\bar{T}_{\text{CPU}}^{\text{IP}-n}$	7.41	17.32	16.98	3.00	0.07	0.05	0.10	0.14	s
$N_z^{\text{IP}-n}$	196	196	262	541	195	195	258	348	—
$\bar{N}_{\text{iterSEC}}^{\text{IP}-n}$	24.78	37.96	22.88	1.78	2.62	2.18	2.32	1.80	—
$N_{\text{eq,NoSEC}}^{\text{IP}-n}$	41	41	44	68	43	43	50	68	—
$N_{\text{ineq,NoSEC}}^{\text{IP}-n}$	197	197	275	1112	195	195	258	348	—
$\bar{N}_{\text{ineq,finalIP}}^{\text{IP}-n}$	226.04	240.49	301.50	1112.88	197.00	196.36	259.56	348.90	—
$P_{\text{conv}}^{\text{IP}-n}$	98	90	100	100	100	100	100	100	%

TABLE II. Normalized average monetary returns in Northern Germany.

	barley $k=0$	rapeseed $k=1$	wheat $k=2$	unit
\bar{r}^k	570	600	750	€/ha

with $u_l^k \sim \mathcal{N}(0,1)$. To analyze computational aspects of the proposed algorithm, we analyzed 50 random data sets, generated as outlined above. The results are summarized in Table I. Several observations can be made. First, *fixing* the number of serviced crops, as for **CApT**- n for $n = 5, \dots, 8$, significantly reduces CPU-time \bar{T}_{CPU} , by (on average) more than two orders of magnitude. The main computational burden stems from an increased number of SEC-iterations. For every additional SEC-iteration, an additional IP with an increased number of SECs has to be solved. IP-2 appeared to have difficulties converging within 200 SEC-iterations in some cases.

We tested problem instances with up to 35 clusters and 700 fields. For $\tilde{k} > 35$, CVXPY started to fail to converge. Real-world practical cases in Northern Germany for the collaboration of larger farms may involve around 100 fields and three depots. A corresponding simulation experiment is visualized in Figure 4. For its solution with 1119 integer variables, 106 equality constraints and between 1119 (for the first IP-solution without any SECs) to 1130 (for the last SEC-iteration) inequality constraints, 8 SEC-iterations were required with a total CPU-time of 2.6s.

VII. CONCLUSION

We presented a flexible framework for the coupling of crop assignment with vehicle routing for harvest planning in agriculture. The discussed problem is relevant since the decision upon crop assignment must be addressed by every farm manager at the beginning of every yearly work-cycle. We compared eight different IP formulations. We found the four cases with enforced inclusion of any crop out of a set of crops to be computationally most efficient. This enforcement is applicable in practice since the list of eligible crops typically is very limited. For large-scale applications where sole IP formulations are not tractable anymore, we proposed a heuristic algorithm combining the IP-formulations with clustering

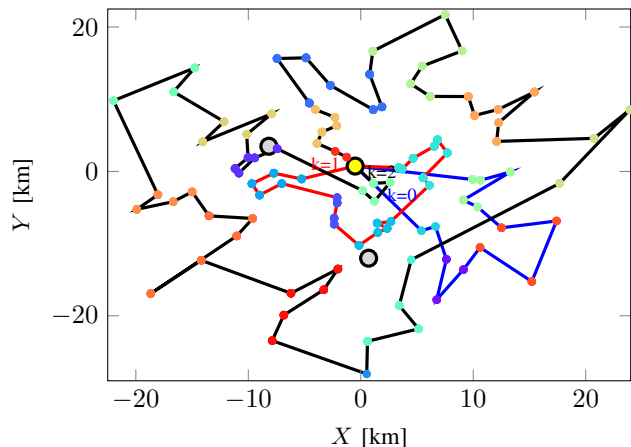


Fig. 4. Visualization of an example for $\tilde{k} = 25$ and $L = 100$ fields. The results of **CApR**-7 are displayed. The inactive and active depots are denoted by the larger gray and yellow balls. The fields are colored according to their clustering result. Labels $k = 0$, $k = 1$ and $k = 2$ indicate the *first* traversal of each crop-tour.

of fields and the solution of local TSPs. To summarize, for practical applications, we thus recommend:

- For reasons of computational efficiency, focus on **CApR**- n for $n = 5, \dots, 8$.
- Increase the number of clusters \tilde{k} as much as the available combination of computational hardware and IP-solver permits in order to reduce the heuristic nature of Algorithm 1.
- Solve Algorithm 2 to determine rates for the renting out and taking of leases on fields.
- In case of a single depot, solve **CApR**-5.
- In case of multiple depots, always solve at least **CApR**-6, and additionally at least one of **CApR**-7 and **CApR**-8 to determine leasing rates for depot usage.
- Put emphasis on detailed parameter modeling according to Step 1 of Algorithm 1, and the inclusion of constraints on *crop rotation* (5) and *traveling time* (12).

Future work will include the testing of alternative IP-solvers including Numberjack [26] and [27], and reformulations of the SECs, for example, in form of MTZ-SECs [28] which introduce additional *continuous* variables for SECs and thereby render the problem of *mixed* integer nature. It is also planned

to test a *Tabu search* heuristic [29]. We stress that limited harvesting time windows with limited harvester traveling speeds naturally constrain the maximum number of fields that can be serviced within a crop-tour. Thus, more efficient IP-solvers are not sought to increase the maximum field number beyond 700, but to increase tractable \tilde{k} and thereby reduce the heuristic nature of Algorithm 1. While this paper focused on the development of a framework for planning at the *beginning* of the yearly work cycle, future work will also revise online coordination of machinery at the *end* of the yearly work cycle during harvest.

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